Title: Implicit kinetic schemes for the Saint-Venant system

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Abstract:

The Saint-Venant system is a vertically averaged hyperbolic model aiming to describe free surface flows in the shallow water regime. To approximate its solutions, we are interested in the design of numerical methods satisfying certain discrete properties inherited from the continuous model, among which the positivity and conservation of the water height, the preservation of lakes at rest as well as the existence of an entropy inequality. Previously an explicit kinetic scheme has been proposed in [2] which satisfies all of the above properties under a CFL condition, however the associated discrete entropy inequality contains an error term which does not always dissipate the energy. Our goal is to improve the stability of this scheme by considering its implicit version.

To this end I will briefly recall the kinetic formalism and the reason why it is well suited to obtain discrete entropy inequalities. I will then focus on the simplified case of flows over a flat bottom, for which we obtain a fully implicit kinetic solver satisfying a discrete entropy inequality and keeping the water height positive without any restriction on the time step. Under a simplification it is possible to express the update analytically, and I will discuss its practical implementation and computational cost. Finally I will detail the handling of varying bottoms through an iterative solver taking advantage of the hydrostatic reconstruction technique [1]. This time a CFL condition is required to get the previous properties but, unlike the explicit scheme, we can ensure that no error term is present in the discrete entropy inequality. These results will be illustrated through numerical experiments.

References:

[1] Emmanuel Audusse et al. "A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows." In: *SIAM J. Sci. Comput.* 25 (2004), pp. 2050–2065.

[2] Emmanuel Audusse et al. "Kinetic entropy inequality and hydrostatic reconstruction scheme for the saint-venant system." In: *Mathematics of Computation* 85 (2016), pp. 2815–2837.

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