

High-order path-conservative methods for non conservative hyperbolic systems based on in-cell discontinuous reconstructions – DR.MOOD generalization

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Abstract

We are interested in the numerical approximation of discontinuous solutions in non conservative hyperbolic systems. Due to the lack of control of numerical viscosity, standard path-conservative methods converge, in general, to weak solutions with wrong jump conditions (see [1]). In [3] a new strategy based on in-cell discontinuous reconstructions which deal with this challenging topic was developed. Next, in [4] the authors extended this strategy to second-order accuracy. In this work we present an extension to high-order accuracy. It is based on the combination of the first-order in-cell discontinuous reconstruction with the MOOD philosophy [2]. The first-order strategy allows in particular to capture exactly the isolated shocks and this new high-order extension keep this property and improve the results when dealing with more than one shock. Several numerical tests are proposed to validate the methods.

Keywords: Non conservative systems, high-order methods, MOOD, shock-capturing methods

References

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