Title: A fully well-balanced hydrodynamic reconstruction

Authors: Christophe Berthon[1] & Victor Michel-Dansac[2]

Abstract:

The present work concerns the numerical approximation of the weak solution of the shallow-water model over a non-flat topography. Here, a particular attention is paid to the steady solutions with nonzero velocity, and the derived scheme has to exactly preserve these stationary solutions, in addition to the commonly preserved lake at rest steady solution.

To address such an issue, an extension of the well-known hydrostatic reconstruction [1] is proposed. Arguing suitable definition of the reconstructed states at the interfaces, any numerical flux function, combined with a relevant source term discretization, produces a well-balanced scheme which exactly preserves the moving and non-moving steady solutions. This eliminates the need to construct specific numerical fluxes such as the one developed in [2] for instance.

In addition, we prove that the resulting scheme is consistent with the homogeneous system on flat topographies, and reduces to the hydrostatic reconstruction when the velocity vanishes. Moreover, in order to increase the accuracy of the simulations, a well-balanced high-order procedure is proposed. Several numerical experiments illustrate the interest of the numerical scheme.

References:

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[1] Université de Nantes, CNRS UMR 6629, Laboratoire de Mathématiques Jean Leray, 2 rue de la Houssinière, BP 92208, 44322 Nantes, France

[2] Université de Strasbourg, CNRS, Inria, IRMA, F-67000 Strasbourg, France