Title: A New Approach for Designing Well-Balanced Schemes for the Shallow Water Equations: A Combination of Conservative and Primitive Formulations

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Abstract: In this paper, we introduce a new approach for constructing robust well-balanced numerical methods for the one-dimensional Saint-Venant system with and without the Manning friction term. Following the idea presented in [R. Abgrall, Commun. Appl. Math. Comput. 5(2023), pp. 370-402], we first combine the conservative and non-conservative (primitive) formulations of the studied conservative hyperbolic system in a natural way. The solution is globally continuous and described by a combination of point values and average values. The point values and average values will then be evolved by two different forms of PDEs: a conservative version of the cell averages and a possibly non-conservative one for the points. We show how to deal with both the conservative and non-conservative forms of PDEs in a well-balanced manner. The developed schemes are capable of exactly preserving both the still-water and moving-water equilibria. Compared with existing well-balanced methods, this new class of scheme is nonlinear-equations-solver-free. This makes the developed schemes less computationally costly and easier to extend to other models. We demonstrate the behavior of the proposed new scheme on several challenging examples.

References:

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