Title: Link between the low Mach number accuracy problem and the solution of a wave system

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Abstract: Classical finite volume schemes for compressible Euler system are not accurate at low Mach number in the sense that they do not converge to the incompressible limit when the Mach number tends to zero [2]. The spurious mode that jeopardizes the convergence can be identified and corresponds to the long time limit of a wave system whose properties and discretization depend on the scheme used for the compressible system [4]. Then, the low Mach number accuracy can be analyse by studying the long time solution of the associated wave system. Applying this methodology to the low Mach fixes of [1, 5], a Godunov scheme with a centered discretization of the pressure gradient is obtained on the associated wave system. Working on this discretization of the wave system, it can be proved that the long time limit exists [3] but could contain oscillating modes (checkerboard modes). Moreover, these oscillating modes can induce a loss of mesh convergence. The existence and the development of these oscillating modes can be explain by the properties of the pressure centered for the wave system.

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