Title: Stable second order boundary conditions for Lattice Boltzmann schemes

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Abstract:

The Lattice Boltzmann Method (LBM) is a versatile approach used to approximate hyperbolic systems of conservation laws. It involves approximating the system using a small set of kinetic equations, which are coupled through a stiff relaxation source term.

To solve these kinetic equations and relaxation source terms, a splitting algorithm is employed. The kinetic equation is solved exactly using the characteristic method, while the relaxation step is approximated using an over-relaxation algorithm with a relaxation parameter, denoted as ω. Setting $\omega = 1$ ensures an entropic scheme, while $\omega = 2$, corresponding to over-relaxation, ensures a second-order scheme.

In this study, we extend the approach to properly handle boundary conditions. The primary challenge arises in spatial dimensions higher than one, where the characteristic structure of the kinetic model significantly differs from that of the original hyperbolic system. Specifically, the number of boundary conditions required for stability generally varies between the hyperbolic model and the kinetic model.

To address this issue, we propose a general approach that ensures both stability and a high-order boundary condition scheme. The theoretical foundation relies on entropy duality results established by Bouchut [2], Aregba *et al.* [1], Dubois [3, 4]. We then apply this method to several classical hyperbolic models to assess its effectiveness. Some of the results we present are contained in the thesis [5].

References

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