Title: Numerical conservative solutions for the Hunter–Saxton equation

Authors: Katrin Grunert[1], Anders Nordli[2], Susanne Solem[3]

Abstract: Solutions of the Hunter–Saxton might enjoy wave breaking in finite time. This means that classical solutions in general do not exist globally, but only locally in time since their spatial derivative might become unbounded from below pointwise in finite time, while the solution itself remains bounded. In addition, energy concentrates on sets of measure zero when wave breaking occurs. As a consequence the prolongation of solutions beyond wave breaking is non-unique.

We will present a numerical method for conservative solutions, i.e., solutions where the energy is not manipulated at breaking time. This method is based on piecewise linear projections, followed by evolution along characteristics where the time step is chosen in order to prevent wave breaking. Convergence is obtained when the time step is proportional to the square root of the spatial step size, which is a milder restriction than the common CFL condition for conservation laws.

References:

K. Grunert, A. Nordli, and S. Solem, Numerical conservative solutions of the Hunter-Saxton equation, BIT 61 (2021), no.2, 441–471.

[1]NTNU Norwegian University of Science and Technology
[2]UiT The Arctic University of Norway
[3]NMBU Norwegian University of Life Sciences