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Explicit, implicit and semi-implicit high-order well-balanced finite volume schemes for general one-dimensional hyperbolic partial differential equations

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Abstract: This work is framed within the context of the numerical resolution of hyperbolic systems of balance laws of the form

$$U_t + F(U)_x = S(U)H_x, \tag{1}$$

where $U(x,t)$ takes value in $\Omega \subset \mathbb{R}^N$, $F : \Omega \to \mathbb{R}^N$ is the flux function; $S : \Omega \to \mathbb{R}^N$; and H is a known continuous function from $\mathbb{R} \to \mathbb{R}$. Since these systems admit non-trivial stationary solutions, it is important that the numerical methods preserve all the stationary solutions, or at least a representative set of them: such methods are known as well-balanced. Some of the authors have previously presented in [1] a general procedure to obtain explicit high-order well-balanced methods based on well-balanced reconstruction operators. Moreover, since in general reconstruction operators are not well-balanced, a methodology to obtain well-balanced reconstruction operator from standard ones was proposed, which requires finding stationary solutions whose cell averages coincide with some given values in the cells. This strategy has been successfully applied to obtain well-balanced schemes for which an explicit or implicit expression of the stationary solutions is available. The objective of this work is the development of general explicit, semi-implicit, and implicit well-balanced finite volume numerical methods, whether or not the expression of the stationary solutions is available, which is achieved through the application of application of Runge-Kutta collocation methods. In particular, the design of implicit and semi-implicit schemes is performed by combining the previous strategy with a time discretization method for the time evolution of type RK-IMEX or RK-implicit (see [2]). This family of well-balanced methods have been applied to several problems, such as the so-called shallow water equations with topography and Manning friction.

Keywords: Systems of balance laws, well-balanced methods, reconstruction operators, implicit methods

References

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