Multilayer modeling of hydrostatic Herschel-Bulkley fluids

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Fluids that may behave as a fluid or a solid have been widely studied in recent years. This is the case of Bingham and Herschel-Bulkley ([1]) fluids. In the viscoplastic models describing these fluids, if the stress of the material is above the yield stress threshold τ_y, they behave like a fluid, and otherwise the material is a rigid body. Describing the vertical structure is essential to reproduce such dynamics. However, it is a known fact that solving the 3D Navier-Stokes equations for free-surface flows is computationally very expensive, and it is even worse in the case of complex rheologies. For this reason, some shallow (or depth-integrated) models have been proposed for these viscoplastic flows. However, they cannot reproduce the complex vertical behavior characterizing these fluids.

In this talk we present hydrostatic models within the multilayer (or layer-averaged) framework ([2]) for yield stress fluids derived from the Navier-Stokes equations, which account for the vertical structure of the fluid. In particular, one of these models takes into account the normal stress contributions, allowing us to recover a pseudoplug layer instead of a purely plug zone. These models use a regularization technique to avoid the singularity when the strain rate vanishes and the material behaves as a solid. We also present a well-balanced discretization of the models for steady states at rest. Some numerical tests will be presented to check the well-balance property of the scheme, and also compare the results of the models with analytical solutions and experimental data. The results of this talk are based on those in [3].

References

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