# Well balanced direct Arbitrary-Lagrangian-Eulerian discontinuous Galerkin schemes on Voronoi meshes with topology changes

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**ABSTRACT**

In this talk we present a novel family of high order accurate numerical schemes for the solution of hyperbolic partial differential equations (PDEs) which combines several geometrical and physical structure preserving properties. Indeed, first, we settle in the Lagrangian framework, where each element of the mesh evolves following as close as possible the local fluid flow, so to *reduce* the *numerical dissipation* at contact waves and moving interfaces and to respect the *Galilean and rotational invariance* of the studied PDEs system. In particular, we choose the direct Arbitrary-Lagrangian-Eulerian setting which, in order to always guarantee the *high quality of the moving mesh*, allows to combine the Lagrangian motion with mesh optimization techniques. The employed Voronoi tessellation is thus regenerated at each time step, the previous one is connected with the new one by space-time control volumes, including hole-like sliver elements in correspondence of topology changes, over which we integrate a space-time divergence form of the original PDEs through a *high order* accurate ADER discontinuous Galerkin (DG) scheme [1, 2]. Mass *conservation* and the respect of the *GCL condition* are guaranteed by construction thanks to the integration over closed control volumes, and *robustness* over shock discontinuities is ensured by the use of an *a posteriori* sub-cell finite volume (FV) limiter. On the top of this effective moving mesh framework, we have also made, for the first time in literature, the full ADER DG scheme with *a posteriori* sub-cell FV limiter *well balanced*, by assuring that any projection, reconstruction and integration procedures were always performed by summing up the exact value of the equilibria plus the high order accurate evolution of the fluctuations [3]. The presentation is closed by a wide set of numerical results, including simulations of Keplerian disks, which demonstrate all the claimed properties and the increased accuracy and robustness of our novel family of schemes.

## References

[1] E. Gaburro, W. Boscheri, S. Chiocchetti, M. Dumbser, C. Klingenberg, V. Springel. *High order direct Arbitrary-Lagrangian-Eulerian schemes on moving Voronoi meshes with topology changes*. Journal of Computational Physics, vol. 407, 109167, 2020.

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[3] E. Gaburro. *High order well balanced Arbitrary-Lagrangian-Eulerian ADER Discontinuous Galerkin scheme on moving Voronoi meshes*. In preparation, 2023.

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