Eigenvalues approximation and numerical treatment of complex regions for two-layer shallow water systems

M.J. Castro[†], **E.D. Fernández-Nieto**[*], **J. Garres-Díaz**[‡], **T. Morales de Luna**[♮]

[†,♮] Dpto. Análisis Matemático, Estadística e Investigación Operativa, y Matemática Aplicada, Universidad de Málaga, Spain (`mjcastro@uma.es, tmorales@uma.es`)
[*] Dpto. Matemática Aplicada I, Universidad de Sevilla, Spain (`edofer@us.es`)
[‡] Dpto. Matemáticas. Universidad de Córdoba, Spain (`jgarres@uco.es`)

ABSTRACT

The two-layer shallow water system is considered to model two non-miscible fluids with different densities under the assumption of hydrostatic pressure. Inside each layer the density is supposed to be constant. Then, the system can be written only in terms of the ratio between both densities. In many applications this ratio of densities, r, is close to 1, such as for example in the case of water with two different concentration of salinity. When $r \approx 1$ an approximation of the eigenvalues of the system was introduced in [7]. They are real except when $|u_1 - u_2|$, the norm of the diffence between the velocities of both layers, are bigger than $\sqrt{g(1-r)(h_1+h_2)}$, being g the constant of gravity, h_1 and h_2 the height of the first and second layer, respectively. Neverthless, by computing exactly the eigenvalues we can observe that the eigenvalues can becomes real if $|u_1 - u_2|$ is bigger enough and the total heigh is smaller. When $r \approx 1$ this is not an usual case in geophysical applications. Nevertheless, by considering the application of the two-layer shallow water model to submarine avalanches (see [3]), we can have this situation for example when an aerial avalanche reachs a lake or a costal area. In this work we investigate firstly a new approximation of the eigenvalues of the system, allowing to approximate this second area of real eigenvalues, when $|u_1 - u_2|$ is bigger than $\sqrt{2g\left(1+\sqrt{r}\right)(h_1+h_2)}$.

Secondly, a second-order Finite Volume solver is proposed to approximate the two-layer shallow water system, named IFCP-FL, as an extention of the solvers introduced in [4] and [2]. It is used as part of a combination of solvers to deals with regions with complex eigenvalues.

Several numerical strategies has been introduced in the bibliography to treat with complex eigenvalues (see [1, 6, 5]). In this work, a new treatment of complex eigenalues is introduced and three different algorithms are compared. Several numerical tests are presented for cases of r close to 1 and 0.3, with applications in fluids and submarine avalanches. In the latter case we show how the treatment based on adding friction are not appropiate.

Keywords: Shallow-Water; Two-layers; Complex eigenvalues; Submarine avalanches; Finite volume schemes, Well-balanced methods; Flux-limiters.

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