Title:

High-order ADER Discontinuous Galerkin schemes for compressible barotropic two-phase flows

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Abstract:

The purpose of this talk is to present a high-order discontinuous Galerkin finite element method to solve the barotropic version of the conservative symmetric hyperbolic and thermodynamically compatible (SHTC) model of compressible two-phase flow, introduced by Romenski *et al.* in [1, 2], in multiple space dimensions.

First, we will show that the original model is only weakly hyperbolic in multiple space dimensions and we will employ two different methodologies to restore strong hyperbolicity: i) the explicit symmetrization of the system, which can be accomplished by adding terms that contain linear combinations of the curl involution, similar to the Godunov-Powell terms in the MHD equations; ii) the use of the hyperbolic generalized Lagrangian multiplier (GLM) curl-cleaning approach forwarded. With both approaches, we obtain a full set of linearly independent eigenvectors, proving that strong hyperbolicity can indeed be restored.

The PDE system is solved using a high-order ADER discontinuous Galerkin method with *a posteriori* sub-cell finite volume limiter to deal with shock waves and the steep gradients in the volume fraction commonly appearing in the solutions of this type of model. We will run several different test cases and benchmark in one and two spatial dimensions, as well as provide the convergence order analysis of the scheme, to demonstrate how well the approach performs. The comparison of the results with those found in the literature, specifically the comparison of one-dimensional results with the exact solution presented in [3] and reference solutions obtained with other well-known models, such as the reduced barotropic Baer–Nunziato-type model, demonstrates the accuracy of the proposed methodology even in the presence of sharp gradients in the solution and an excellent agreement in all cases, as can be seen in [4].

References

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[4] Río-Martín, L., Dumbser, M.: High-order ADER Discontinuous Galerkin schemes for a symmetric hyperbolic model of compressible barotropic two-fluid flows, 2023 (Submitted).