

Entropy stable, positivity preserving and well-balanced Godunov-type schemes for multidimensional shallow water system

Alessia Del Grosso^{††1}, Agnes Chan^{† ‡}, Gérard Gallice[†], Raphaël Loubère[‡] and Pierre-Henri Maire[†]

[†] CEA Cesta, Le Barp, France,

[‡] Université de Bordeaux, Bordeaux INP, CNRS, LRC Anabase, Bordeaux,

ABSTRACT

A class of cell centered finite volume schemes has been introduced to discretize the equations of Lagrangian hydrodynamics on moving mesh [4]. In this framework, the numerical fluxes are evaluated by means of an approximate Riemann solver, based at the nodes of the mesh, which provides the nodal velocity required to move the mesh in a compatible manner. In this presentation, we describe the generalization of this type of discretization to hyperbolic systems of balance laws written in Eulerian representation. The evaluation of the numerical fluxes relies on an approximate Riemann solver located at the mesh nodes. The construction of this nodal solver uses the Lagrange-to-Euler transformation introduced by Gallice [3] and revisited in [1, 2] to build positive and entropic Eulerian Riemann solvers from their Lagrangian counterparts. The application of this formalism to the case of gas dynamics provides a positive and entropic finite volume scheme under an explicit condition on the time step. In this work, we extend this scheme to handle source terms, in particular for the shallow-water system of equations. We show how to render the scheme well-balanced in 1D and 2D. The numerical assessment of this scheme by means of representative test cases will be presented for the first and second orders. In particular, the good behavior is illustrated by the absence of the typical numerical pathology of traditional finite volume approaches for such system of PDEs.

References

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¹Contact: alessia.del-grosso@math.u-bordeaux.fr