Efficient implicit numerical schemes for the multimaterial Euler equations in Lagrangian coordinates

S. Chiocchetti[1], G. Russo[2], S. Boscarino[2]
simone.chiocchetti@unitn.it, russo@dmi.unict.it, boscarino@dmi.unict.it

ABSTRACT

The computation of approximate solutions to the multimaterial Euler equations poses several challenges, first and foremost the issue of artificial smearing of material parameters across interface boundaries. Lagrangian schemes completely eliminate this issue, but at the cost of rather stringent timestep restrictions.

In this work we introduce an implicit numerical method for the multimaterial Euler equations in Lagrangian coordinates. The implicit discretization is aimed at bypassing the prohibitive timestep restrictions present in flows with stratified media, where one of the materials is particularly dense, or rigid (or, even worse, both). This is the case for flows of water-air mixtures, air-granular media, or similar high density ratio systems.

We will present the novel discretization approach, which makes extensive use of the remarkable structure of the governing equations in Lagrangian coordinates to find the solution by means of a single implicit discrete wave equation for the pressure field, yielding a symmetric positive definite structure and thus a particularly efficient algorithm. Additionally, we will discuss filtering strategies for contrasting the emergence of pressure or density oscillations typically encountered in multimaterial flows, and will present results concerning the robustness, accuracy, and performance of the proposed method, including applications to stratified media with high density and stiffness ratios.

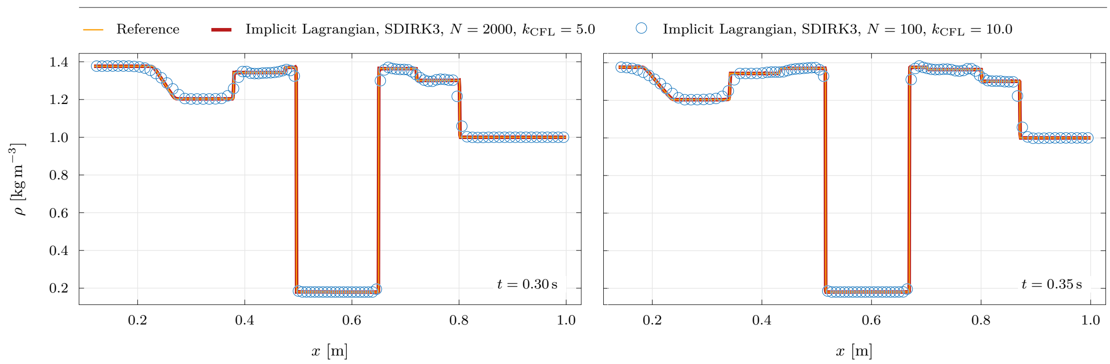

Figure 1: One-dimensional shock–bubble interaction problem. The solution obtained with the proposed implicit Lagrangian scheme and third order diagonally implicit Runge–Kutta timestepping, on $N = 2000$ uniformly spaced mesh points (i.e. with non-uniform mass Δm) at $k_{\mathrm{CFL}} = 5.0$, and with $N = 100$ points at $k_{\mathrm{CFL}} = 10$.

S. Chiocchetti acknowledges the support obtained by the Deutsche Forschungsgemeinschaft (DFG) via the project DROPIT, grant no. GRK 2160/2, and from the European Union's Horizon Europe Research and Innovation Programme under the Marie Skłodowska-Curie Postdoctoral Fellowship MoMeNTUM, grant agreement No. 101109532.

[1]Institute of Aerodynamics and Gasdynamics, University of Stuttgart, Germany
[2]Department of Mathematics and Informatics, University of Catania, Italy