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Title: A Lagrangian-Eulerian Approach for Multidimensional Systems of Conservation/Balance Laws

Abstract: *In this work, we will discuss a novel Lagrangian-Eurelian procedure for numerical approximation of conservation/balance laws, which is based on an improved concept of no-flow curves/surfaces as recently introduced in [1,2] in a simple two-step manner: The first Lagrangian evolution step automatically handles the nonautonomous hyperbolic fluxes, and the second step Eulerian remap step allows the use of a single grid, thus eliminating the need for moving meshes while retaining local conservation. This new methodology admits fully-discrete [3,4] and semi-discrete [5,6] formulations, along with numerical analysis via weak asymptotic analysis [7,8]: convergence for the unique entropy (Kruzhkov) solution to scalar hyperbolic problems [4,5] and a positive principle to the more general case of multi-D systems [6]. This Lagrangian-Eulerian framework has been shown to be suited very well to address typical challenges in the numerical solution of hyperbolic conservation (and balance) laws with discontinuous nonlinearity and systems with irregular solutions. The method is Riemann-solver-free and, hence, time-consuming field-by-field type decompositions are avoided in the case of systems and, due to the no-flow curves/surfaces, there is no need to employ/compute the eigenvalues (exact or approximate values) of the relevant Jacobian of the numerical flux functions, and thus giving rise to an effective weak CFL-stability condition which is feasible in the computing practice. This method has been successfully applied for solving nontrivial hyperbolic problems in 1D and Multi-D (scalar and systems), e.g., 2D shallow water equations with variable topography and discontinuous data in a geometric intrinsic formulation [9], compressible Euler Flows and conservation laws with discontinuous space-time dependent flux functions [10], Orszag-Tang vortex system and a nonstrictly hyperbolic three-phase flow system (with a resonance point) [6]. This scheme has also been successfully applied to nonlocal hyperbolic problems [11,12]. We will also provide numerical 1D/Multi-D examples to verify the theory and discuss/illustrate the capabilities of the proposed approach.*

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